# Topological Combinatorics (Oberseminar)

Ongoing Research Seminar

## Description

In this seminar we treat current research problems in discrete and computational geometry that can be approached using methods from algebraic topology. The aim of the seminar is to make advances in research and introduce students to potential research topics.

All seminars take place in the seminar room of the "Villa", Arnimallee 2, 14195 Berlin.

## Speakers

| Speaker | Date | Time | Title and Abstract |
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| Aldo Guzmán Sáenz (Cinvestav, MX) | Oct 22, 2014 | 5 PM s.t. | Title: **The cohomology ring away from 2 of F(RP^n,k)**<br>Abstract: In this talk we present a computation of the cohomology ring of the configuration space of *k* ordered points in the *n*-th dimensional real projective space. The strategy used for this computation allows us to compute the cohomology ring of ordered configurations in the punctured *n*-th dimensional real projective space. |
| Daniela Egas Santander<br><br>(U Bonn) | Oct 29, 2014 | 5:15 PM | Title: **On the homology of Sullivan Diagrams**<br><br>Abstract: In string topology one studies the algebraic structures of the chains of the free loop space of a manifold by defining operations on them. Recent results show that these operations are parametrized by certain graph complexes that compute the homology of compatifications of the Moduli space of Riemann surfaces. Finding non-trivial homology classes of these compactifications is related to finding non-trivial string operations. However, the homology of these complexes is largely unknown.   In this talk I will describe one of these complexes: the chain complex of Sullivan diagrams. In the genus zero case, I'll give a reinterpretation of it in terms of weighted partitions, give some computational results, and if time permits, I'll describe work in progress that suggests that these are highly connected. This talk is based on joint work with F. Lutz. |
| Alexander Gaifullin<br><br>(IITP Moscow) | Nov 05, 2014 | 5PM s.t. | Title: **Flexible polyhedra and their volumes**<br>Abstract: Consider a closed polyhedral surface in the Euclidean three-space with rigid faces and with hinges at edges. If this polyhedral surface admits a deformation (a flexion) that is not induced by an ambient rotation of the space, then it is called a *flexible polyhedron*. A definition of a flexible polyhedron in an n-dimensional Euclidean space is completely similar. A problem on existence of flexible polyhedra turned out to be rather non-trivial. First examples of flexible polyhedra are Bricard's self-intersected flexible octahedra (1897). However, the first example of an embedded (i.e., non-self-intersected) flexible polyhedron was constructed by Connelly only in 1977. One of the most amazing facts in the theory of flexible polyhedra is Sabitov's theorem claiming that the volume of an arbitrary flexible polyhedron in the three-dimensional Euclidean space is constant during the flexion. (Earlier this assertion was known as the Bellows Conjecture.)<br><br>The talk will contain a survey of recent results by the speaker concerning flexible polyhedra, and the main ideas behind these results. This will include:  (1) The constructions of self-intersected flexible cross-polytopes in Euclidean and Lobachevsky spaces of all dimensions, and of embedded flexible cross-polytopes in spheres of all dimensions. In dimensions 5 and higher these are the first known examples of flexible polyhedra and in dimensions 4 and higher these are the first known examples of embedded flexible polyhedra.  (2) The proof of the analogue of the Bellows Conjecture in the Euclidean spaces of arbitrary dimensions.  (3) The proof of the analogue of the Bellows Conjecture in odd-dimensional Lobachevsky spaces, and the counterexamples to the Bellows Conjecture in spheres of all dimensions.  (4) Results on the polyhedral relations among the entries of the Gram matrix of the period vectors for a doubly-periodic two-dimensional polyhedral surface in the three-dimensional Euclidean space. |
| -- | Nov 12, 2014 | | (no talk) |

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| Micha Laso<br><br>(FU Berlin) | Nov 19, 2014 | 5PM s.t. | Title: **Obstacles for splitting multidimensional necklaces**<br><br>Abstract: "The well-known “necklace splitting theorem” of Alon asserts that every k-colored necklace can be fairly split into q parts using at most t cuts, provided k (q  1)  t. In a joint paper with Alon et al. we studied a kind of opposite question. Namely, for which values of k and t there is ameasurable k-coloring of the real line such that no interval has a fair splitting into 2 parts with at most t cuts?<br><br>We proved that k > t + 2 is a sucientcondition (while k > t is necessary).We generalize this result to Euclidean spaces of arbitrary dimension d, and to arbitrary number of parts q. We prove that if k(q  1) > t + d + q  1,then there is a measurable k-coloring of R^d such that no axis-aligned cube has a fair q-splitting using at most t axis-aligned hyperplane cuts. Our bound is of the same order as a necessary condition k(q  1) > t. Moreover, for d = 1, q = 2 we get exactly the previous result.<br><br>Additionally, we prove that if a stronger inequality k(q1) > dt+d+q1 is satised, then there is a measurable k-coloring of R^d with no axis-alignedcube having a fair q-splitting using at most t arbitrary hyperplane cuts. The proofs are based on the topological Baire category theorem and use algebraic independence over suitable chosen elds." |
| Florian Frick<br><br>(TU Berlin) | Nov 26, 2014 | 5PM s.t. | Title: **Constraining with equivariant maps: Barycenters of polytope skeleta**<br>Abstract: By the van KampenFlores theorem the d-skeleton of the (2d+2)-simplex is not embeddable into Euclidean space of dimension 2d. The closely related ConwayGordonSachs theorem states that for any embedding of the complete graph on six vertices in 3-space there are two vertex-disjoint linked triangles. Recently, Dobbins proved that any point in an nd-polytope is the barycenter of n points in the d-faces of the polytope. We will give simple, mostly combinatorial proofs of these three results that all build on the same idea: constraining points in a cell complex to an appropriate subcomplex by exploiting symmetry and the pigeonhole principle. Moreover, we will remark on generalizations. This is joint work with Pavle V. M. Blagojevi and Günter M. Ziegler. |
| -- | Dec 3, 2014 | -- | (no talk) |
| Pavle Blagojevi<br><br>(FU Berlin) | Dec 10, 2014 | 5 PM s.t. | Title: **Topology of the Grünbaum hyperplane mass partition problem**<br><br>**Abstract** |
| Marko Berghoff<br><br>(HU Berlin) | Dec 17, 2014 | 5PM s.t. | Title: **Wonderful Renormalization**<br><br>Abstract: This talk is about how so-called wonderful compactifications can be used to solve an extension problem for distributions appearing in quantum field theory. This extension problem is one variant of what physicists call renormalization, a collective term for various ways of extracting physical sensible quantities out of a priori ill-defined integrals arising in perturbative calculations. Roughly speaking, physics associates to a given graph G (representing an element in the perturbative expansion of some physical quantity) a distribution that is defined only outside of a subspace arrangement determined by G; renormalization then amounts to extending this distribution onto this arrangement - this is where wonderful compactifications enter the game as a way to systematically reduce the problem to a toy model case. These compactifications were first introducedby DeConcini and Procesi in the case of linear arrangements, based on ideas from Fulton and MacPherson's famous article "A Compactification of Configuration Spaces". What makes them so well-suited for this extension problem is the fact that both the wonderful construction as well as renormalization in general, are governed by the underlying combinatorial structure. This structure is encoded in a certain subset of the poset of all subgraphs of G and allows to describe the problem's solution in purely combinatorial terms (once some initial data is fixed). I will quickly sketch how the problem emerges in physics and describe its solution. Then I show how this geometric/combinatorial approach allows us to study the ambiguity of extensions obtained in this way. This leads to the renormalization group, a powerful tool that even allows for statements beyond perturbation theory (i.e. about the "real world"). |

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| Jose Rodriguez<br><br>(Notre Dame) | Jan 7, 2015 | 5PM s.t. | Title: **The maximum likelihood degree and data discriminants of likelihood equations**<br>Abstract: The maximum likelihood degree (ML degree) is a topological invariant of a variety and in the nicest cases the ML-degree is a signed Euler characteristic. The ML degree measures the algebraic complexity of the maximum likelihood estimation problem in algebraic statistics, which is my motivation for computing these numbers. The first part of the talk will provide an introduction to the ML degree and its connections to topological Euler characteristics. The second part of the talk will offer computational results followed by conjectures. |
| Isaac Mabillard<br><br>(IST Austria) | Jan 14, 2015 | 5PM s.t. | Title: **Eliminating Tverberg Points: An Analogue of the Whitney Trick**<br><br>Abstract: Kuratowski's Theorem gives a criterion to decide the planarity of a graph. I.e., whether a 1-simplicial complex can be embedded in R^2. This problem can be generalised to the embeddability of a n-simplicial complex K into R^{2n}, and this question is solved by van Kampen embeddability criterion: the vanishing of an obstruction cocycle is linked to the existence of an embedding. This readily yields a polynomial-time algorithm for deciding embeddability. The proof of this result is based on the Whitney trick: if two n-balls intersect in R^{2n} in two points of opposite signs, then one can "remove" these two intersections by a "local" isotopy.<br><br>In my talk, I will explain how this trick also works for configurations with more than two balls. In my drawings, "more than two" is going to mean "three". For instance, three balls intersecting in two points of opposite signs can be "untangled".  More generally, the Whitney trick also works for intersection points of higher multiplicity.<br><br>This fact leads to a generalised version of the van Kampen Criterion for the existence of maps K -> R^d without self-intersection of "high multiplicity". In particular, it shows that the problem "Does a complex K mapping into R^d has a "Tverberg-type" theorem?" is decidable in polynomial time -- but we must stress that our techniques only work if the dimension of K is at most d-3. |
| Mimi Tsuruga<br><br>(TU Berlin) | Jan 21, 2015 | 5PM s.t. | Title: **Improving bistellar simplification**<br><br>Abstract: Over a century ago, a mining engineer who dabbled a bit in mathematics and physics, too, conjectured that a 3-manifold having both the homology and fundamental group of a sphere must also be homeomorphic to the 3-sphere. This so-called (3-dimensional) Poincar\'e conjecture can be generalized to topological d-manifolds and homeomorphisms between them. And rather than the TOP category, we could instead think about the PL category, where we would work with PL manifolds and PL homeomorphisms between them. While the topological Poincar\'e conjecture has been proven in all dimensions, the PL version is still open in dimension 4; it is referred to as the smooth Poincar\'e conjecture in dimension 4 (or SPC4) since PL and DIFF coincide in dimension 4.<br><br>In the PL category, we can take advantage of the combinatorial nature of the objects at hand and consider a computational problem often referred to as sphere (or manifold) recognition. In dimension 3, the computational complexity of sphere recognition is in NP and the problem is unrecognizable (ie, undecidable) in dimension 5 and up, while the complexity of this decision problem in dimension 4 remains open.<br><br>We present our construction of one family of difficult-to-recognize 4-spheres and the heuristic algorithms that we used in an attempt to recognize them to indeed be spheres. One algorithm which will be discussed has considerable room for improvement; it uses Udo Pachner's bistellar flips. |

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| Daniel Lütgehetmann<br><br>(FU Berlin) | Jan 28, 2015 | 5PM s.t. | Title: **Configuration Spaces of Graphs**<br><br>Abstract: The i-th rational cohomology of the n-th ordered configuration space of a (nice enough) topological manifold (for any fixed i and n) satisfies representation stability, a concept introduced by Benson Farb and Thomas Church which generalizes homological stability. This implies, for example, that given any such manifold we can calculate this cohomology for all n >> 0 simultaneously by a finite calculation. Furthermore, the dimension of this i-th cohomology is eventually a polynomial in n, the number of particles.<br><br>If we instead of manifolds look at graphs, the situation is more complicated: the dimension of the corresponding cohomology for a graph grows much faster than polynomially in some degrees, even in the simplest cases. To investigate this cohomology, it is useful to construct a deformation retraction of these configuration spaces with a CW structure. In this talk, we will do this explicitly for any locally finite graph. This allows us to compute the cohomology explicitly in a few cases, but the general case is still unknown. If the graph we are considering is finite, then this CW complex will also be finite, which allowed us to use it for calculations with a computer; if there is time we will present some of them. |
| Steven Simon<br><br>(Wellesley College) | Mar 26, 2015 | 5PM s.t. | Title: **Average-Value Tverberg Partitions via Finite Fourier Analysis**<br><br>Abstract: The topological Tverberg conjecture claimed, for any continuous map from the boundary of a N(q,d):=(q-1)(d+1)-simplex to d-dimensional Euclidian space, the existence of q pairwise disjoint faces whose images have non-empty q-fold intersection. The affine cases, true for all q, constitute Tverberg's famous generalization of Radon's theorem on partitioning point collections into disjoint sets with overlapping convex hulls. Although the conjecture was established for all prime powers in 1987 by \"Ozaydin, counterexamples for all non-prime-powers were shown to exist in 2015 by Frick. Reformulating this conjecture in terms of finite harmonic analysis and considering maps below the tight dimension N(q,d), we show that one can nonetheless guarantee collections of q pairwise disjoint faces -- including when q is not a prime power -- which satisfy a variety of ``average value" coincidences arising from the vanishing of Fourier transforms. |
| William Browder<br><br>(Princeton) | June 3, 2015<br><br>June 10, 2015 | 4.15PM<br><br>4.15PM | Title: **The Cohomology of G-Spaces (G compact)**<br><br>**Abstract** |
| Pablo Soberon Bravo<br><br>(U Michigan) | July 1, 2015 | 4.15PM | Title: **Variations of positive-fraction intersection results in combinatorial geometry**<br><br>Abstract: We discuss conditions on families of sets in R^d which allow us to obtain "positive fraction" intersection results or give upper bounds for the piercing number of the family. In particular we show existence results for weak epsilon-nets for families of alpha-lenses and extensions of Helly-type theorems with volumetric conditions. |
| Felix Jonathan Boes<br><br>(U Bonn) | July 2, 2015 | 4.15PM | Title: **The Moduli space of Riemann Surfaces**<br><br>Abstract: In order to classify the complex structures on a given two dimensional manifold F, one introduces the moduli space of Riemann surfaces which we denote by M. Its underlying points are in one-to-one correspondence to the equivalence classes of complex structures on F. The classification of complex structures is then the following question: What is the (co)homology of M? The talk is divided into two parts.<br><br>1) I will report on certain aspects of M, e.g. I will motivate its occurence in string topology<br><br>2) There is a nice cellular model for M provided by Bödigheimer. I will discuss this model in some detail. It allows us to perform calculations, i.e. we can answer the above question partially. |

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| Djordje Baralic<br><br>(MI SANU, Belgrade) | July 2, 2015 | 5.00PM | Title: **Toric topology of n-colorable simple polytopes and applications**<br><br>Abstract: We present recent results about topology and combinatorics of quasitoric manifolds and small covers. Quasitoric manifolds are topological analogue of the nonsingular projective toric variety and we describe their construction from a simple polytope P^n and a characteristic map. The cohomology ring and the characteristic classes of these manifolds is calculated by Davis and Januszkiewicz and combinatorics of the polytope and its Stanley-Reisner ring are crucial for their description. Also, we address several interesting applications of specially constructed quasitoric manifolds over simple polytopes with low chromatic number.  Following and developing ideas of R. Karasev, we extend the Lebesgue theorem (on covers of the cubes) and the Knaster-Kuratowski-Mazurkiewicz theorem (on covers of the simplices) to different classes of convex polytopes (colored in the sense of M. Joswig). We also show that the n-dimensional Hex theorem admits a generalization where the n-dimensional cube is replaced by a n-colorable simple polytope. We prove several theorems about embeddings and immersions  of these manifolds into Euclidean  spaces. The use of specially designed quasitoric manifolds, with easily computable cohomology rings and the cohomological cup-length, offers a great flexibility and versatility in applying the general method. |
| Peter Patzt<br><br>(FU Berlin) | Oct 21, 2015 | 4.15PM | Title: **Representation Stability**<br><br>An interactive introduction to the topic. |
| Daniel Lütgehetmann<br><br>(FU Berlin) | Oct 28, 2015 | 4.15PM | Title: **The Totaro Spectral Sequence**<br><br>A short introduction to sheaf cohomology and using the Totaro spectral sequence to deduce representation stability of configuration spaces of manifolds. |

# Organizers

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