Berlin Mathematical School

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Discrete Geometry 1 – Problem Sheet 3

Please hand in your solutions to Prof. Ziegler on **Wednesday, Nov. 6, 2013** before the lecture begins. Please put your name and student ID (if you have one) on the first page of your solutions and staple the sheets together.

Problem 1: *Posets* (4 Points)

Let $(Q, \leq)$ be a finite partially ordered set (poset). Show that any two of the following three properties yield the third property:

(i) $(Q, \leq)$ is bounded.

(ii) For any two elements $a, b \in Q$ their join $a \vee b$ exists.

(iii) For any two elements $a, b \in Q$ their meet $a \wedge b$ exists.

Problem 2: *Faces and Lattices* (4+4 Points)

Recall that the set of faces F of a polytope P together with the inclusion relation "$\subseteq$" is a finite graded lattice that satisfies the diamond property.

(a) Give an example of a finite graded lattice satisfying the diamond property that is not the face lattice of a polytope. Give an explanation.

(b) Let P be a polytope with vertex set V. Prove that any subset of V defines a unique face of P if and only if P is a simplex.

Problem 3: *Example of a Face Lattice* (2+2+4(+2) Points)

Consider the example of a face lattice $L(P)$ of a polytope P with vertices $V = \{v_0, \ldots, v_9\}$ that Professor Ziegler handed out in class on Wednesday. It can be found on the course website under "Lecture Notes Etc.".

Answer the following questions and explain how you came up with your answers.

(a) How many vertices do the 2-faces of P have?

(b) How many facets do the 3-faces of P have?

(c) Choose three vertices $u, v, w \in V$ of P and determine the number of vertices that the vertex figures $P/u, P/v$ and P/w have.

(d) *Bonus:* Choose a vertex Figure P/v for $v \in V$ and draw a polytope that has the same combinatorial type as P/v. (The *combinatorial type* of a polytope is the equivalence class of its face lattice under isomorphism.)