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Discrete Geometry 1 – Problem Sheet 10

Please hand in your solutions to Prof. Ziegler on **Wednesday, Jan. 15, 2014** before the lecture begins.

Problem 1: *Calculate an h-Vector* (6 Points)

Let C_d^Δ denote the d-dimensional crosspolytope. Find and describe a suitable shelling of $\mathcal{C}(\partial C_d^\Delta)$ and compute the h-vector of C_d^Δ.

Problem 2: *Dehn-Sommerville Equations* (2+2 Points)

Let P be a simplicial d-polytope.

(a) Check that the Dehn-Sommerville equations for $d = 4$ are equivalent to the two linear relations $f_0(P) - f_1(P) + f_2(P) - f_3(P) = 0$ and $f_2(P) = 2f_3(P)$.

(b) For $d = 5$, find a linear relation that follows from the Dehn-Sommerville equations but is independent of the Euler-Poincaré relation and $2f_3(P) = 5f_4(P)$.

Problem 3: *f-Vectors of Neighborly Polytopes* (2+2+2 Points)

(a) Compute the f-vector of the cyclic polytope $C_4(7)$.

(b) Prove the following statement: The f-vectors of all neighborly simplicial polytopes of dimension d on n vertices are identical.

(c) What is the maximal possible number of vertices in a simple 5-polytope with 8 facets?

Problem 4: *Shellings for the Cube* (3+3 Points)

(a) How many different shellings are there for the (boundary of the) 3-dimensional cube C_3?

(b) Show (using induction on the dimension d) that a facet ordering $F_1, \ldots, F_{2d}$ of the d-cube C_d is *not* a shelling if and only if $F_1, \ldots, F_{2j}$ consists of j pairs of opposite facets, for some j, $1 \leq j < d$.