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Discrete Geometry 1 – Problem Sheet 12

Please hand in your solutions to Prof. Ziegler on **Wednesday, Jan. 29, 2014** before the lecture begins.

Problem 1: *Points in Convex Position* (2+6 Points)

Recall that a set $X \subset \mathbb{R}^d$ is *in convex position* if for every $x \in X$ we have $x \notin \operatorname{conv}(X \backslash \{x\})$.

(a) Find a configuration of 8 points in general position in the plane such that *no* 5 of its points are in convex position. Hence you are showing that the "Erdős–Szekeres number" $n(5) > 8$.

(b) Prove that for each $k \geq 1$ there exists a number $m(k)$ such that any $m(k)$ points in the plane contain k points in convex position or k points on a line.

Problem 2: *Dual Configurations* (4+4 Points)

(a) Translate the Sylvester–Gallai Theorem into the dual setting of line arrangements. (In your statement, be careful about parallel lines etc.)

(b) Translate the Erdős–Szekeres Theorem into the dual setting of line arrangements. What is the dual statement to existence of k-caps or k-cups in every sufficiently large point configuration in general position?

Problem 3: *Hyperplane Arrangements* (4(+6) Points)

(a) Consider the arrangement $\mathcal{H}$ of hyperplanes given by the equations $x_i = x_j$ for $1 \leq i < j \leq d$. Draw a picture of $\mathcal{H}$ in dimension 3. How many d-dimensional cells does $\mathcal{H}$ have?

(b) *Bonus:* Consider the arrangement $\mathcal{H}'$ of hyperplanes given by the equations $x_i = \pm x_j$ for $1 \leq i < j \leq d$. Draw a picture of $\mathcal{H}'$ in dimensions 2 and 3. How many d-dimensional cells does $\mathcal{H}'$ have?