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Topologie II – Revision

Date: **Sunday, Feb. 22, 2015**. This is what an exam *could* look like. These problems do not cover all topics of the lecture. Hence the actual exam may cover different topics. Its length will be 90 or 120 mins. The following six problems are intended for 90 mins.

Problem 1: (2 Points)

State the Splitting Lemma and prove one equivalence.

Problem 2: (4 Points)

Compute the homology of real projective space of dimension n denoted by $\mathbb{R}P^n$ for all $n \geq 1$.

Problem 3: (3 Points)

Given a topological space X and a subspace A such that A is a deformation retract of an open neighborhood U in X. Show that $H_n(X, A) \cong \widetilde{H}_n(X/A)$ for all $n \in \mathbb{Z}$.

Problem 4: (4 Points)

(a) Define *degree*.

(b) Construct a surjective map $S^n \longrightarrow S^n$ of degree zero for every $n \geq 1$.

Problem 5: (3 Points)

(a) Define *free resolution*.

(b) Compute a free resolution of $\mathbb{Z}_2$.

(c) Compute $\operatorname{ext}^1_{\mathbb{Z}}(\mathbb{Z}_2 \oplus \mathbb{Z}, \mathbb{Z})$ and $\operatorname{ext}^1_{\mathbb{Z}}(\mathbb{Z}_2 \oplus \mathbb{Z}, \mathbb{Z}_2)$.

Please turn page for an additional problem.

Problem 6: (4 Points)

Let $X = S^1 \times S^1$ be the product of two 1-dimensional spheres.

(a) Compute the homology of X with coefficients in $\mathbb{Z}$, $\mathbb{Z}_2$ and $\mathbb{Q}$.

(b) Compute the cohomology of X with coefficents in $\mathbb{Z}$, $\mathbb{Z}_2$ and $\mathbb{Q}$.

Note: You may not use existing formulas for the singular homology of $S^1 \times S^1$ without proof.