Berlin Mathematical School

Freie Universität Berlin

Prof. Pavle Blagojević
Albert Haase
Prof. Holger Reich

Arbeitsgruppe Diskrete Geometrie

Arbeitsgruppe Algebraische Topologie

Topologie II – Exercise Sheet 4

Date of assignment: **Wednesday, Nov. 26, 2014**. We highly recommend problems marked with a star.

*Exercise 1: *Deformation Retractions*

(a) Show that if A is a retract of X, meaning there exists a retraction $r\colon X \longrightarrow A$, then the maps

$$H_n(A) \longrightarrow H_n(X)$$

in homology induced by the inclusion $A \hookrightarrow X$ are injective.

(b) Give an example of a space that is contractible but does not deformation retract to a point.

(c) The following topological space, up to homeomorphism, is called the *Möbius strip*:

$$M := [0,1] \times [0,1]/\sim,$$

where $\sim$ is the equivalence relation generated by $(0,t) \sim (1,1-t)$. Show that M deformation retracts to a circle. By circle we mean a space homeomorphic to S^1.

Exercise 2: *Open Maps*

Show that a continuous map from a compact space to a Hausdorff space is open, that is, sends open sets to open sets.

*Exercise 3: *Topology of Simplicial Complexes*

(a) Give an example of a geometric simplicial complex in some $\mathbb{R}^d$ whose topology does not agree with the subspace topology.

(b) Show that a simplicial complex is a Hausdorff space.

(c) Show that a simplicial complex is compact if and only if it is finite, meaning it is a set of only finitely many simplices.

*Exercise 4: *Simplicial Homology*

(a) Compute the simplicial homology of a wedge of two circles directly. Extend your reasoning and model to compute the simplicial homology of the wedge of two spheres $S^k \vee S^\ell$.

(b) Calculate the simplicial homology of the möbius strip M directly. *Note:* Not every triagulation of the square will lead to a triangulation of M!

(c) How could we have calculated the homology of M more easily?