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Topologie II – Exercise Sheet 4

Date of assignment: **Wednesday, Nov. 26, 2014**. We highly recommend problems marked with a star.

*Exercise 1: *Deformation Retractions*

- (a) Show that if A is a retract of X , meaning there exists a retraction $r: X \rightarrow A$, then the maps

$$H_n(A) \rightarrow H_n(X)$$

in homology induced by the inclusion $A \hookrightarrow X$ are injective.

- (b) Give an example of a space that is contractible but does not deformation retract to a point.
- (c) The following topological space, up to homeomorphism, is called the *Möbius strip*:

$$M := [0, 1] \times [0, 1] / \sim,$$

where \sim is the equivalence relation generated by $(0, t) \sim (1, 1 - t)$. Show that M deformation retracts to a circle. By circle we mean a space homeomorphic to S^1 .

Exercise 2: *Open Maps*

Show that a continuous *and bijective* map from a compact space to a Hausdorff space is open, that is, sends open sets to open sets.

*Exercise 3: *Topology of Simplicial Complexes*

- (a) Give an example of a geometric simplicial complex in some \mathbb{R}^d whose topology does not agree with the subspace topology.
- (b) Show that a simplicial complex is a Hausdorff space.
- (c) Show that a simplicial complex is compact if and only if it is finite, meaning it is a set of only finitely many simplices.

***Exercise 4:** *Simplicial Homology*

- (a) Compute the simplicial homology of a wedge of two circles directly. Extend your reasoning and model to compute the simplicial homology of the wedge of two spheres $S^k \vee S^\ell$.
- (b) Calculate the simplicial homology of the möbius strip M directly. *Note:* Not every triangulation of the square will lead to a triangulation of M !
- (c) How could we have calculated the homology of M more easily?