Exercise Sheet for *Topology I*, 2017/18

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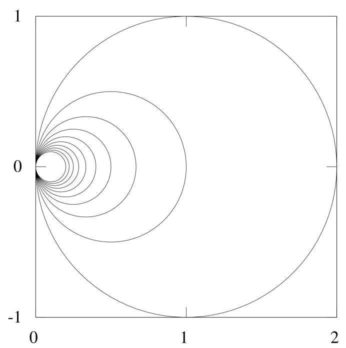
Sheet 10 due Wednesday, January 17th, 2018



The ten largest circles of the Hawaiian earring.

Exercise 37 (Compact Hausdorff implies normal)

Let X be a compact Hausdorff space. Prove that X is normal.

Exercise 38 (Hawaiian earring)

Consider $X \subset \mathbb{R}^2$ given by the union of all circles with center $(\frac{1}{n}, 0)$ and radius $\frac{1}{n}$ for $n = 1, 2, \dots$. Show that X is not an Euclidean neighborhood retract.

Show that X is not an Euclidean neighborhood retract regardless of the inclusion into $\mathbb{R}^d$

(The last part might be a bit more difficult. Knowledge about retracts and fundamental groups can be useful. $\pi_1(S^1) \neq 0$ may be assumed.)

Exercise 39 (Homotopy of pairs)

Give an example of two continuous maps of pairs $f, g\colon (X, A) \to (Y, B)$ such that the induced maps $f_X, g_X\colon X \to Y$ are homotopic and the induced maps $f_A, g_A\colon A \to B$ are homotopic, but f and g are **not** homotopic as maps of pairs.

Exercise 40 (Compactly generated topology)

Given a topological space X. Denote by kX the set X with the finest topology such that for all compact Hausdorff spaces K and continuous maps $f\colon K \to X$ the induced set map $f_*\colon K \to kX$ is continuous as well.

A space X is called *compactly generated* when $kX = X$.

1. Prove that the topology on kX is given by: $A \subset kX$ is closed if $f_*^{-1}(A)$ is closed for every K compact Hausdorff and every $f\colon K \to X$ continuous.
2. Give an example of a compactly generated space that is not Hausdorff.
3. Show that every quotient of a compact Hausdorff space is compactly generated.
4. Show that every quotient of a compactly generated space is again compactly generated.
5. Let $X = \mathbb{R}$ with the cocountable topology. Show that X not compactly generated. The cocountable topology is given by: $A \subset \mathbb{R}$ is closed if A is countable or $A = \mathbb{R}$. (Hint: What are compact subsets of X.) What is kX?