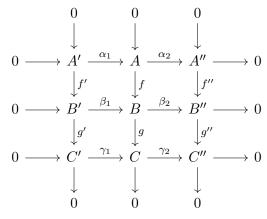
Exercise Sheet for Topology II, 18

Prof. Pavle Blagojević, Jonathan Kliem

Sheet 5

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Exercise 20 (More diagram chasing) Let R be a ring and consider the following commutative diagram of R-modules:



Assume it is known, that all the rows and columns except for

$$0 \to B' \xrightarrow{\beta_1} B \xrightarrow{\beta_2} B'' \to 0$$

are exact. Assume further that im $\beta_1 \subset \ker \beta_2$ or that $\ker \beta_2 \subset \operatorname{im} \beta_1$. Prove that all rows and columns are exact!

Exercise 21 Let $h_* = (h_n, \partial_n^h)_{n \in \mathbb{Z}}$ be a homology theory satisfying axioms (1)–(4) and let Y be a topological space. Show that for suitably defined ∂_n^k we obtain a new homology theory k_* satisfying axiom (1)–(3) be setting

$$k_n(X, A) = h_n(X, A) \oplus h_n(X \times Y, A \times Y).$$

If h_* satisfies axiom (5) (additivity), so does k_* . If Y is contractible, then k_* satisfies (4) (so k_* is ordinary).

Exercise 22 Let h_* be a homology theory satisfying axioms (1)–(5) with coefficients in \mathbb{Z} . Let X be a finite graph with X_0 the set of vertices.

1. For all $x \in X$ and all $n \in \mathbb{Z}$ show that $h_n(X, X \setminus \{x\})$ is a finitely generated free \mathbb{Z} -module and define the valence of x as

$$\operatorname{val}(x) = \operatorname{rk}_{\mathbb{Z}} h_1(X, X \setminus \{x\}) + 1.$$

- 2. If $x \notin X_0$ then val(x) = 2.
- 3. A homeomorphism $f \colon X \to Y$ is valence preserving, i.e., it sends points of valence k to points of valence k.