Exercise Sheet for *Topology II*, 18

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Sheet 6 Discussion: Wednesday, May 30th, 2018

Exercise 23 Let h_* be a homology theory satisfying axioms (1)–(5). Let $n \geq 0$ and let

$$a\colon S^{2n} \to S^{2n}, x \mapsto -x$$

be the antipodal map. Show that a induces multiplications by -1 on the reduced homology group $\tilde{h}_{2n}(S^{2n})$.

Exercise 24 Let h_* be a homology theory satisfying axioms (1)–(5) with coefficients in G, where $\operatorname{char} G \neq 2$. Let $n \geq 0$ and let $f\colon S^{2n} \to S^{2n}$ be a map without fixed points. Show that f is not homotopic to the identity.

Exercise 25 Let h_* be a homology theory satisfying axioms (1)–(5). Let X, Y be spaces and $(X_1, \dots, X_n)$ an open cover of X and $(Y_1, \dots, Y_n)$ an open cover of Y.

Let $f\colon X \to Y$ be a a map such that $f(X_i) \subset Y_i$ for all $i \in \{1, \dots, n\} = [n]$ and such that for all $\emptyset \neq I \subset [n]$ the map

$$f_I\colon \bigcap_{i \in I} X_i \to \bigcap_{i \in I} Y_i$$

given by the restriction of f induces and isomorphism $h_*(f_I)$. Show that also

$$h_*(f)\colon h_*(X) \to h_*(Y)$$

is an isomorphism.

Exercise 26 Let h_* be a homology theory satisfying axioms (1)–(5) and let X be a topological space.

1. Show that
$$h_k(X \times S^n, X \times \{x_o\}) \cong h_{k-1}(X \times S^{n-1}, X \times \{x_0\})$$
for all $k \in \mathbb{Z}$, $n \geq 1$ and $x_0 \in S^{n-1} \subset S^n$.
2. Show that
$$h_k(X \times S^n) \cong h_k(X) \oplus h_k(X \times S^n, X \times \{x_0\})$$
for all $k \in \mathbb{Z}, n \geq 0$ and $x_0 \in S^n$.
3. Deduce that
$$h_k(X \times S^n) \cong h_k(X) \oplus h_{k-n}(X)$$
for all $k \in \mathbb{Z}$ and all $n \geq 0$.