Exercise Sheet for *Topology II*, 18

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Sheet 7 Discussion: Wednesday, June 13th, 2018

Exercise 27 Suppose A and B are open subset of X and $C \subset A \cap B$. Show that there are two relative versions of the long exact Mayer-Vietoris sequences:

$$\dots \to H_k(X, A \cap B) \to H_k(X, A) \oplus H_k(X, B) \to H_k(X, A \cup B) \to H_{k-1}(X, A \cap B) \to \dots$$

$$\dots \to H_k(A \cap B, C) \to H_k(A, C) \oplus H_k(B, C) \to H_k(A \cup B, C) \to H_{k-1}(A \cap B, C) \to \dots$$

Exercise 28 Let $\sigma \colon \Delta^1 \to \mathbb{R}$ be given by $\sigma(t_0, t_1) = 1 - 2t_0$. Show that this respresents a generator of $H_1(\mathbb{R}, \mathbb{R} \backslash \{0\})$ (which we know to be isomorphic to $\mathbb{Z}$).

Exercise 29 (Checking the requirements of Mayer-Vietoris) Let $f \colon \mathbb{R} \to \mathbb{R}$ be the (non-continuous) function

$$f(x) = \begin{cases} \sin(\frac{1}{x}), & x > 0 \\ 0, & x \leq 0 \end{cases}$$

Let further $X_- = \{(x, y) \in \mathbb{R}^2 | y \leq f(x)\}$ and $X_+ = \{(x, y) \in \mathbb{R}^2 | y > f(x)\}$.

Show that the pair X_-, X_+ does not induce a Mayer-Vietoris sequence.

Exercise 30 Let h_* be a homology theory satisfying axioms (1)–(5) and let $(X_i, x_i)_{i \in I}$ be a collection of pointed spaces, where points are closed, such that each x_i has an open neighborhood U_i such that $h_k(U_i, \{x_i\}) \to h_k(X_i, \{x_i\})$ is the zero map for all k

1. Show that $\bigvee_{i \in I}$, the one-point union is the sum in the category of pointed spaces.
2. Show that for every k there is a natural isomorphism

$$\bigoplus_{i \in I} h_k(X_i, \{x_i\}) \xrightarrow{\cong} h_k(\bigvee_{i \in I} X_i, \{x\}),$$

where x is the base point of the wedge.