Exercise Sheet for *Topology II*, 18

Prof. Pavle Blagojević, Jonathan Kliem

Sheet 8 Discussion: Wednesday, June 20th, 2018

Exercise 31 Compute the homology of Σ_g the oriented surface of genus g, using the Mayer-Vietoris sequence.

It might be usefull to remember, how we calculated the fundamental group of Σ_g in Exercise 58 in Topology I (`https://wikis.fu-berlin.de/display/discgeom/Topology+I`])

Exercise 32 Compute the homology of a Klein bottle.

Compute the homoloy of the product of Klein bottle and $\mathbb{RP}^2$.

Compute the homology of the product of Klein bottle and $\mathbb{RP}^n$.

Exercise 33 Compute the homology of $\mathbb{RP}^n \times \mathbb{RP}^m$ for $n, m \geq 0$.

Exercise 34 Let $X = S^1 \cup_f D^2$ and $Y = S^1 \cup_g D^2$ where $f\colon S^1 \to S^1$ has degree p and $g\colon S^1 \to S^1$ has degree q. Compute the homology of $X \times Y$.

(Let $f\colon A \to X$, where $A \subset Y$. $X \cup_f Y$ denotes the adjunction space, that is the quotient space of $S^1 \sqcup D^2$ obtained by identifying $x \in A \subset Y$ with $f(x) \in X$.)